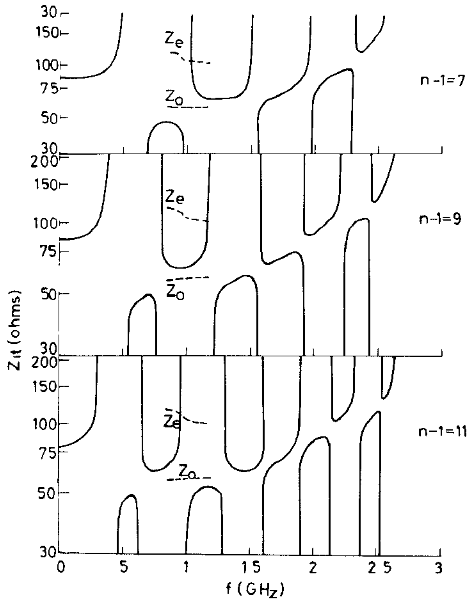

Fig. 7. The iterative impedance of an odd number $(n-1)$ of lines. Z_e and Z_o of one line-pair of the same configuration are also shown for comparison by dashed lines.

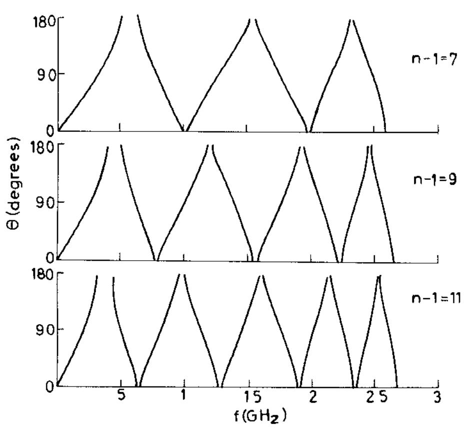

Fig. 8. The phase shift due to an odd number $(n-1)$ of lines.

Z_{it} and θ with frequency for 7, 9, and 11 lines. Around 17.5 GHz, all the curves have a passband; however, the Z_{it} characteristics for $3,7,11,\cdots$ lines differ markedly from those for $5,9,\cdots$ lines. In some passbands Z_{it} is nearly constant and phase shift is linear with frequency. With an increase in the number of lines, more stopbands appear and shift towards lower frequencies.

VI. Conclusion

Iterative impedance Z_{it}, and phase shift θ, (assuming negligible attenuation) have been calculated for even (8, 10, and 12), and odd (7, 9, and 11) coupled lines, and their variations with frequency have been compared. When all possible couplings are taken into account, it was found that the unit cell approximation is inadequate. The stopbands increase in number, become narrower, and shift towards lower frequencies as the number of lines is increased. For even pairs of lines, the number of stopbands increases and the phase changes at a slower rate when compared to the case of odd-pairs of lines. When the number of lines is odd, there is a clear difference between the Z_{it} curves for the groups of $3,7,11$ lines and $5,9$ lines. All the curves have a passband around 17.5 GHz. In some passbands, Z_{it} is uniform and θ is linear with frequency.

This method can be used for other periodic structures as well, e.g., interdigital line, etc., and enables determination of precise locations of stopbands.

Acknowledgment

The author wishes to express his thanks to Dr. K. N. Laxminarayan of the Physics Department, Panjab University, Chandigarh, India, and to Dr. Bharathi Bhatt, Centre for Applied Research in Electronics, Indian Institute of Technology, Delhi, India, for their valuable suggestions.

References

[1] E. M. T. Jones and J. T. Bolljahn, "Coupled-Strip-Transmission-Line Filters and Directional Couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.
[2] W. M. Libbey, "Characteristics of a microstrip two-meander ferrite phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 483–487, July 1973.
[3] J. A. Weiss, "Dispersion and field analysis of a microstrip meander-line slow-wave structure, "*IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 1194–1201, Dec. 1974.
[4] R. Crampagne and M. Ahmadpanah, "Meander and interdigital lines as periodic slow-wave structures. Part I. Characteristics of waves propagating along an infinite array," *Int. J. Electron.*, vol. 43, no. 1, pp. 19–32, July 1977.
[5] R. Crampagne and M. Ahmadpanah, "Meander and interdigital lines as periodic slow-wave structures. Part II. Applications to slow-wave structures." *Int. J. Electron.*, vol. 43, no. 1, pp. 33–39, July 1977.

The Bandwidth of Image Guide

RICHARD. J. COLLIER and ROBIN D. BIRCH

***Abstract*—The various parameters involved in the bandwidth of image guide are discussed, viz., the aspect ratio and dielectric constant. Three definitions of bandwidth are given involving dispersion, wave-guiding properties and variation of characteristic impedance with velocity. Theoretical values of these definitions are given and the paper concludes with a discussion about their relative importance.**

I. Introduction

A large amount of work is being carried out into the uses of image line. At frequencies where better known wave guiding structures experience difficulties, by suitable choice of parameters, image lines can be made which are less susceptible to many of these problems. This work has been both theoretical [1]–[3], [10] and practical [4], [5] but little consideration has been paid to what actually are the practical constraints limiting the bandwidth available to image line. Without due consideration, it might appear that since the fundamental mode exists down to zero hertz the bandwidth is purely and simply the cutoff frequency of the next higher order mode. This is far from the case and this paper will discuss the major problems limiting the bandwidth. Topics such as dispersion, wave guiding properties and radiation will be dealt with, along with a discussion as to the usable upper frequency limit.

Manuscript received December 28, 1979; revised March 19, 1980.
The authors are with the Electronics Laboratories, University of Kent at Canterbury, Canterbury, Kent CT2 7NT, England.

0018-9480/80/0800-0932$00.75 ©1980 IEEE

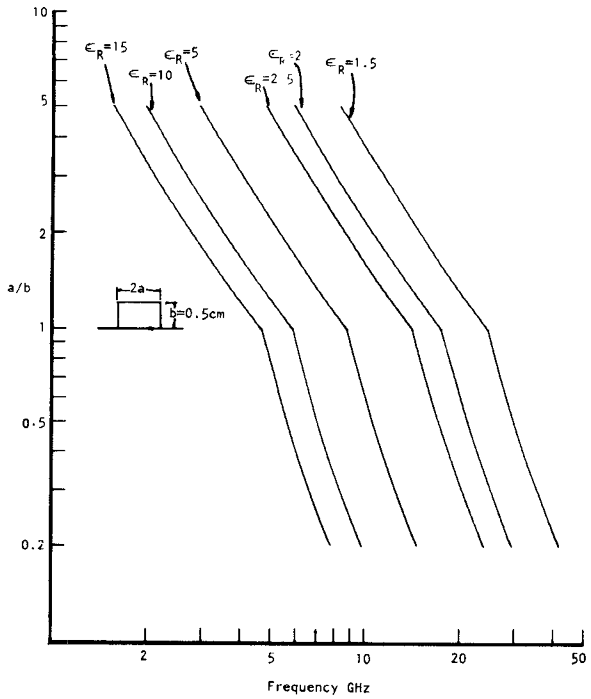

Fig. 1. Cutoff frequencies of the first higher order modes for different aspect ratios and dielectric constants in image guide.

II. The Upper Frequency Limit

As we increase the operating frequency in addition to the fundamental mode there is also a frequency at which higher order modes are liable to be propagated.

Fig. 1 shows these cutoff frequencies for different dielectric constants and various aspect ratios using the theory developed in [2]. For a large aspect ratio the first higher order mode to appear will be the E_{12}^x mode (i.e., a single maxima in the horizontal direction and a double maxima in the vertical direction). If the aspect ratio is small the first mode to appear will be the E_{21}^x mode (i.e., a double maxima in the horizontal direction and a single maxima in the vertical direction). At unity aspect ratio these two modes are degenerate.

It is worth noting that image line can be frequency scaled. In Fig. 1 the dimension B is 5 mm, by reducing this by a factor of 5, we also increase the frequency scale by a factor of 5.

III. Lower Frequency Limit–Dispersion Conditions

Since image line does not support purely TEM modes, the propagation along the line is dispersive. We can define an arbitrary limit on this dispersion in terms of the percentage change of phase velocity within the band. This then gives a lower usable frequency at which the phase velocity is chosen to have changed by 20 percent from its value at the next higher order mode. From this defined limit the *dispersion bandwidth* follows. Fig. 2 shows a set of *dispersion bandwidths* for various aspect ratios and dielectric constants. It can be seen that for a given aspect ratio greater bandwidths are obtained as the dielectric constant decreases. A direct comparison of bandwidth can be made between the various aspect ratios and dielectric constants due to all the values being normalized to the cutoff frequency for the guide with unity aspect ratio and a dielectric constant of 2.5 (i.e., 14.53 GHz). Fig. 3 shows what these guides physically look like after being normalized to have the same cutoff frequency.

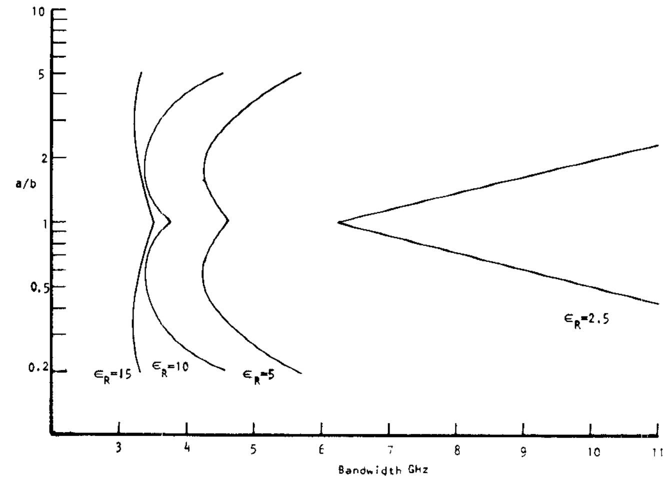

Fig. 2. Dispersion bandwidth for various aspect ratios and dielectric constants.

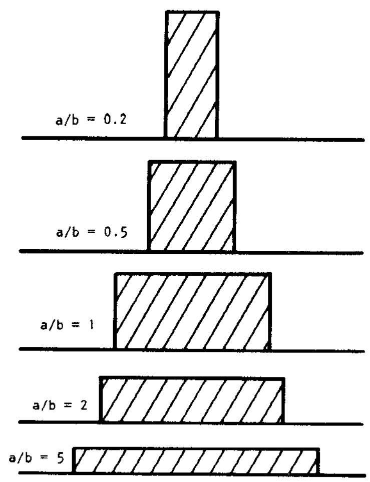

Fig. 3. Cross sections of image guide with different aspect ratios but the same cutoff frequency for the first higher order mode.

IV. Lower Frequency Limit–Wave Guiding Properties

For a given aspect ratio and dielectric constant the amount of energy travelling in the air and the amount travelling inside the dielectric is dependent on frequency. As the frequency increases, the travelling wave becomes better confined to the dielectric. The net effect of this is for the wave to become less likely to radiate from discontinuities. Because the power ratio between air and dielectric is dependent on frequency we can define a lower frequency point by limiting the amount of power allowed to travel in air as compared to dielectric. This ratio can be limited to a maximum value of unity at the lower frequency point. The following equations for fields in the dielectric and air regions of the image line were used by the authors to calculate the power ratio

$$E=E_0\cos\left(\frac{\psi' x}{a}\right)\cos\left(\frac{\psi' y}{b}\right)\exp\left(-j\beta z\right),\qquad \text{inside the dielectric}$$

$$E=E_0\cos\left(\psi'\right)K_0(\tau r)\exp\left(-j\beta z\right),\qquad \text{outside the dielectric.}$$

For most of the air outside the guide the evanescent electric fields are cylindrical in nature. The solution of Maxwell's equa-

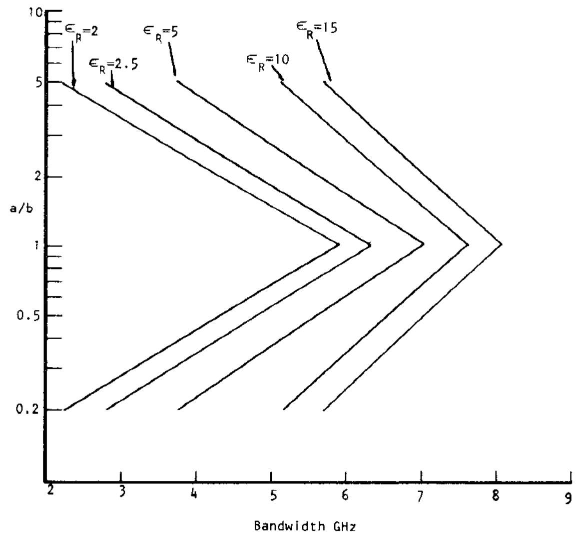

Fig. 4. Wave-guiding bandwidth for various aspect ratios and dielectric constants.

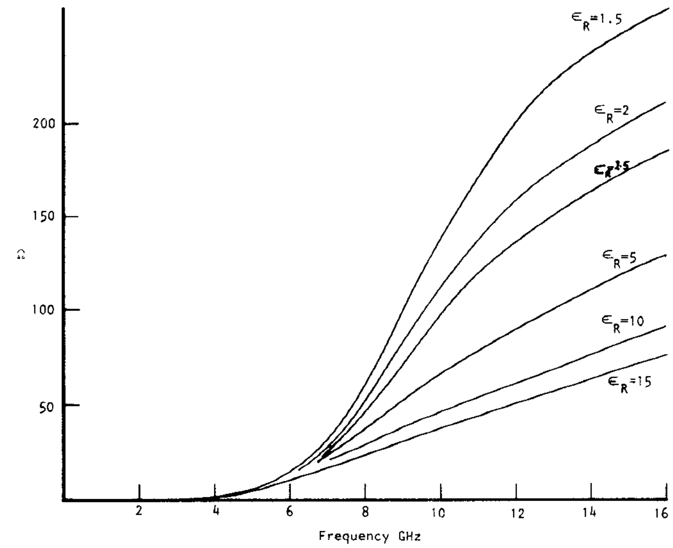

Fig. 6. Variation of Z_0 with frequency for unity aspect ratio and different dielectric constants.

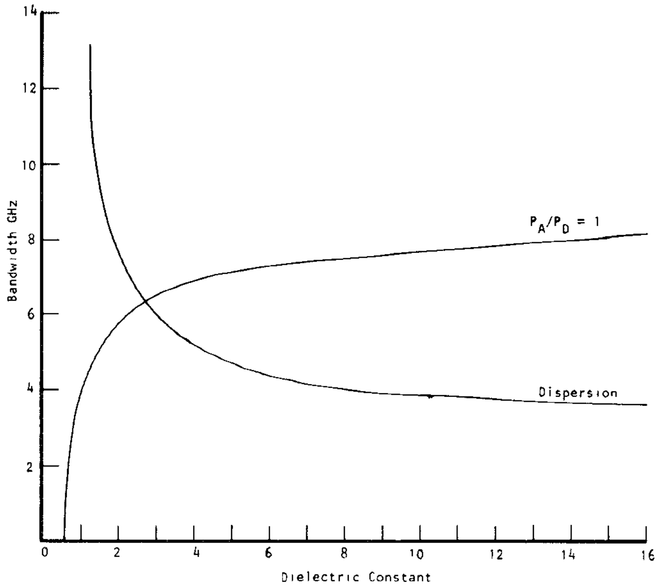

Fig. 5. Comparison between the dispersion bandwidth and the wave-guiding bandwidth for unity aspect ratio and diferent dielectric constants.

tions for these fields will contain modified Hankel functions. $K_0(\tau r)$ is a zero order modified Hankel function, valid for the far field and has been matched to experimental results in the near field within a few percent. By substituting the above equations into the equation for the ratio of power in air and dielectric, vis.

$$\frac{\text{Power in Air}}{\text{Power in Dielectric}} = \frac{\int_A \bar{E} \times \bar{H}^* ds}{\int_D \bar{E} \times \bar{H}^* ds} = \frac{\int_A |E|^2 ds}{\int_D \sqrt{\epsilon_R}\, |E|^2 ds}.$$

A frequency can be found at which this ratio is unity. Fig. 4 shows the available bandwidth calculated from this frequency for various aspect ratios and dielectric constants. Increased bandwidths are obtained with increasing dielectric constant. For each dielectric constant value there is a maximum bandwidth when the aspect ratio approaches unity. Again these results are normalized as before to an upper frequency of 14.53 GHz. Fig. 5 shows the combined results for the two contraints of bandwidth so far discussed. From this figure the "optimum" dielectric constant with unity aspect ratio would be ~2.5. Obviously if the two definitions for the lower usable frequency were changed the two plots would intersect at a different place and hence a different "optimum" dielectric constant would result.

Several authors have discussed radiation from image guide. Linear image guide does not have any leaky modes and therefore is free from radiation losses [7]. On the other hand, curved image guide does radiate and various authors have given definitions for the minimum acceptable radius of curvature [6].

It would be a difficult task to attempt to define bandwidth in terms of the radiation losses from bends since there are too many variables which would have to be fixed by definition, *viz.*, ratio of radius of curvature to width of image guide, angle of bend, and minimum acceptable attenuation. It is the opinion of the authors that by choosing a sufficiently large value of the radius of curvature the full bandwidth described in this paper is possible.

The other source of radiation losses in image guide is caused by discontinuities [8] and these have to be carefully designed in order to keep such losses to an acceptable level.

V. Low Frequency Limit–Variation of Characteristic Impedance

The Characteristic Impedance of Image Guide has not, as far as the authors are aware, been defined for image guide. The authors suggest the following definition may prove useful in subsequent circuit design [9]:

$$Z_0 = \frac{b^2 E^2}{2(P_A + P_D)} \Omega$$

where E is the maximum value of the electric field. Fig. 6 shows the variation of Z_0 with frequency for various dielectric constant. It can be seen in that figure that due to the similar shape of the curves the bandwidth in terms of percentage change in Z_0 is independent of the dielectric constant. These curves have been plotted for a unity aspect ratio. If a 50-percent variation of Z_0 can be tolerated throughout the band then the lower frequency limit will be 9.35 GHz, i.e., a bandwidth of 5.18 GHz.

Conclusion

The practical conditions have been discussed which limit available bandwidth. From the single requirement of low dispersion we have the largest bandwidth when dealing with non unity aspect ratios and low dielectric constant. In contrast, to obtain good wave guiding ability and low radiation from discontinuities a higher dielectric constant is required. If only one of these conditions is critical we can adjust the parameters of the guide to suit that particular requirement and obtain the largest bandwidth. If all the conditions are equally important we are limited to a small range of dielectric constants, e.g., for the contraints discussed an image line with unit aspect ratio has the maximum bandwidth at $\epsilon_R = 2.5$

We have used for the upper limit the first higher order mode cutoff frequency. In practice it seems that it is possible to maintain a single mode operation slightly above this frequency since the first higher order mode has extensive external fields at its cutoff frequency and is unlikely to be propagated in any circuit until a frequency is sufficiently high enough to reduce the evanescent fields down to a manageable size.

The purpose of this paper has been to give some guidance about the many parameters involved in the bandwidth of image guide in order to help in the design of broad-band circuits.

References

[1] S. P. Schlesinger and D. D. King, "Dielectric image lines," *IRE Trans.* pp. 291–299, July 1958.

[2] R. M. Knox and P. P. Toulios, "Integrated circuits for the millimetre through optical frequency range," *Proc. Symp. Submillimetre Waves*, New York, NY, 1970.

[3] K. Ogusu, "Numerical analysis of the rectangular dielectric waveguide and its modifications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT- 25, Nov. 1977, pp. 874–885.

[4] J. A. Paul and Yu-Wen Chang, "Millimetre-wave image guide integrated passive devices," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 751–754, 1978.

[5] D. D. King, "Circuit components in dielectric image guide," *IRE Trans. Microwave Theory Tech.*, Dec. 1955, pp. 35–39.

[6] K. Solbach, "The measurement of the radiation losses in dielectric image line bends and the calculation of a minimum acceptable curvature radius," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, Jan. 1979, pp. 51–53.

[7] A. A. Oliver and S. T. Ping, "A new class of leaky modes on open dielectric waveguides," in 1979 *IEEE* MTT-S Int. Microwave Symp. (Orlando, FL), pp. 569–571.

[8] K. Solbach, "Grating-tuner in dielectric image line for integrated millimetre wave circuits," in *Proc. 9th European Microwave Conf.*, Brighton, England, Sept. 17–20, 1979, pp. 458–462.

[9] R. D. Birch and R. J. Collier, "Study of image guide bandwidth," in *Proc. 9th European Microwave Conf.*, Brighton, England, 1979, pp. 453–457.

[10] E. A. J. Marcatilli, "Dielectric rectangular waveguide and directional coupler for integrated optics." *Bell System Tech. J.*, vol. 8, no. 7.

Tolerance Analysis of Cascaded Structures

J. W. BANDLER, FELLOW, IEEE, AND M. R. M. RIZK, MEMBER, IEEE

***Abstract*—This paper presents an analysis scheme to obtain the response of a cascaded network and its first-order sensitivities wrt design variables at the vertices of the tolerance region in an efficient and systematic way. This information is needed in worst-case search algorithms to identify the worst vertex or in a general tolerance assignment. A substantial saving in computational effort is achieved by using the new approach over the basic approach of reanalyzing the circuit at every vertex.**

Manuscript received December 5, 1979; revised March 19, 1980. This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

The authors are with the Group on Simulation, Optimization and Control and Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

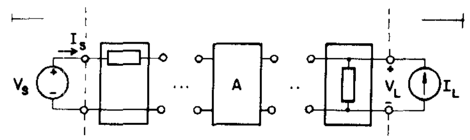

Fig. 1. Cascaded network with appropriate terminations.

I. Introduction

A recent approach for the analysis of cascaded networks (using the chain matrix) has been used efficiently to perform response evaluation as well as simultaneous and arbitrary large-change sensitivity evaluation [1]. This paper presents an analysis and sensitivity evaluation scheme using the recent approach in a form suitable for tolerance analysis of such networks.

In tolerance assignment each parameter ϕ has a tolerance associated with it so that it can have a value lying between $\phi^0+\epsilon$ and $\phi^0-\epsilon$, where ϕ^0 is the nominal value and ϵ is the tolerance. The number of vertices of the tolerance region is 2^k, where k is the number of the toleranced parameters, which includes all different combinations of parameter values.

The tolerances on elements as well as the nominal parameter values are optimized, consequently the response and its first-order sensitivity at the vertices of the tolerance region [2] are needed by the optimization algorithms. This information is particularly useful if a worst-case search algorithm [3] has to identify the worst vertex [4], [5].

A specific algorithm designed for evaluating the response and its sensitivities at the vertices of a tolerance region is presented. An example is given along with a comparison between the new approach and the conventional way (the reanalysis) for evaluating the response and its sensitivities at the vertices.

II. Necessary Background

The analysis approach is based on two types of analyses. The first is the forward analysis which consists of initializing a $\bar{u}^T$ row vector as either e_1^T, e_2^T, where

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

or a suitable linear combination and successively premultiplying each constant chain matrix by the resulting row vector until an element of interest or a termination is reached. The second is the reverse analysis, which is similar to conventional analysis of cascaded networks, proceeds by initializing a v column vector as either e_1, e_2 or a suitable linear combination and successively postmultiplying each constant matrix by the resulting column vector, again until either an element of interest or a termination is reached.

Consider the network in Fig. 1. Applying forward and reverse analyses up to A, we obtain the expression

$$Q \triangleq \bar{u}^T A v \qquad (1)$$

where

A transmission or chain matrix of the element of interest,

$\bar{u}$ vector obtained from forward analysis (initiated at the source and ending at the input port of A),

v vector obtained from reverse analysis (initiated at the load and ending at the output port of A).